LCA Methodology

Application of Markov Chain Model to Calculate the Average Number of Times of Use of a Material in Society

An Allocation Methodology for Open-Loop Recycling

Part 1: Methodology Development

Hiroyuki Yamada[1], Ichiro Daigo[1], Yasunari Matsuno[1*], Yoshihiro Adachi[1] and Yasushi Kondo[2]

[1] Graduate School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
[2] Faculty of Political Science and Economics, Waseda University, 1-6-1 Nishi-waseda, Shinjuku-ku, Tokyo, 169-8050 Japan

* **Corresponding author** (matsuno@material.t.u-tokyo.ac.jp)

Preamble. In this series of two papers, a methodology to calculate the average number of times a material is used in a society from cradle to grave is presented and applied to allocation of environmental impact of virgin material. **Part 1** focuses on methodology development and shows how the methodology works with hypothetical examples of material flows. **Part 2** (DOI: http://dx.doi.org/10.1065/lca2006.05.246.2) presents case studies for steel recycling in Japan, in which the methodology is applied and allocation of environmental impact of virgin steel is conducted.

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Abstract

Goal, Scope and Background. It has been recognized that LCA has a limitation in assessing open cycle recycling of materials because of inevitable subjective judgments in setting system boundary. According with the enforcement of recycling laws, there has been a rapid increase in recycling ratio of materials at the end-of-life of products in many industrialized countries. So, materials' life cycle is getting more complicated, which makes it difficult to quantify the environmental impacts of materials used in a product in an appropriate way. The purpose of this paper is to develop a methodology to calculate the average number of times a material is used in a society from cradle to grave. The method developed in this paper derives the average number of times material is used; this value could be used for allocation of environmental burdens of virgin material as well as an indicator for assessing the state of material use in a certain year, based on material flow of material in that year.

Main Features. Our methodology is based on Markov chain model using matrix-based numerical analysis. A major feature of this method is that it creates transition probability matrices for a material from the way in which the material is produced, consumed, and recycled, making it possible to simply elicit indicators that assess the status of material use in products in society. Our methodology could be an alternative method to derive the average number of times material is used, which could be used for allocation of environmental burdens of virgin material.

Results and Discussions. The methodology was applied to hypothetical examples of material flows, in which a virgin material was produced and used in products, recycled and finally landfilled. In some cases, closed loop and open loop recycling of materials existed. The transition probability matrix was created for each material flow, and how many times a virgin material is used in products until all of the elements are ultimately landfilled.

Conclusions. This methodology is applicable to a complicated material flow if the status of residence of a material and its flow in a society can be figured out. All the necessary data are the amount of virgin material production, amount of the material used in products, recycling rate of the material at the end of life of each product, the amount of scrap of the material that are used for products. In Part 2 of this paper, case studies for steel were conducted.

Keywords: Markov chain; material flow; number of times of use; open loop recycling management

Introduction

A traditional problem in LCA is how to deal with processes where recycled material is used as an input or where the output of a process is further used as raw material in another product system (open-loop recycling). Allocation is needed to partition the environmental impacts caused by the virgin material production, the recycling and the final disposal processes of a material over different product systems.

A critical review was conducted by Ekvall and Finnveden concerning Allocation in ISO 14040, in which the procedure for allocation in LCI was explained as follows [1]. 'The International Organization for Standardization (ISO) presented a standard for LCI-ISO 14041, requiring the following procedure be used for allocation in multifunction processes.

1) Allocation should be avoided, wherever possible, either through division of the multifunction process into subprocesses, and collection of separate data for each subprocess, or through expansion of the systems investigated until the same functions are delivered by all systems compared.
2) Where allocation cannot be avoided, the allocation should reflect the physical relationships between the environmental burdens and the functions, i.e., how the burdens are changed by quantitative changes in the functions delivered by the system.
3) Where such physical casual relationship alone cannot be used as the basis for allocation, the allocation should reflect other relationships between the environmental burdens and the functions.

For allocation in open-loop recycling, ISO 14041 recommends the same procedure but allows a few additional options. If the recycling does not cause a change in the inherent properties of the material, the allocation may be avoided through calculating the environmental burdens as if the material was recycled back into the same product. Otherwise, the allocation can be based on physical properties, economic value, or the number of subsequent uses of the recy-

cled material. The international standards do not include information on the effect of the different methods on the life cycle modeling, for example the feasibility of the methods, the amount of work required, or what type of information that results from the application of the methods'.

Many different solutions to the allocation problems have been suggested and case studies have been conducted. Ekvall et al. [1,2] surveyed published LCA case studies, including open-loop recycling, where allocation was applied. They took into account the feasibility of the different allocation methods and the amount of work required to apply them. One of their conclusions is that an allocation problem can be avoided through system expansion as long as there is an alternative way of generating the exported functions and data can be obtained for this alternative production. It is an adequate way of dealing with allocation when an action will affect an exported function, if the data uncertainties are not too large, and if the indirect effects are important enough to be significant for a decision.

Werner et al. [3] conducted a case study for aluminum window frame, in which system expansion was adopted to allocate the environmental impacts of primary aluminum production between primary and secondary aluminum products. In their case study, an economic allocation procedure for aluminum was developed based on different market prices for secondary materials with different alloy content. In this case, market prices were assumed to reflect the functionality of a material quality within a techno-economic system. In other words, it was assumed that market prices represented the qualitative description of the degradation of a material over a product system. Based on this qualitative degradation, a 'relative resource consumption' were defined. On the other hand, it was stated that this allocation procedure requires a stable price relation of primary and secondary material.

In the case of steel recycling in Japan, the material flow of steel scraps are far more complicated because steels are used for various products, and the recycling ratio of steel scraps from post consumer products varies according to products. Steel scraps from post consumer products are consumed not only for electric arc furnace steel (secondary products) but also for producing converter steel (virgin materials). In some cases, electric arc furnace steels (secondary products) are used as resources for converter steel (virgin materials), which means that 'upgrading' of steel products happens. In addition, the prices of converter steels, electric arc furnace steels as well as steel scraps have changed significantly during this decade. So, one cannot simply define the 'cascade' recycling of steel scraps. Economic allocation seems not so much appropriate for steel scrap recycling.

Then the allocation of environmental burdens of converter steel (virgin material) could be based on the number of subsequent uses of the recycled material. This method was first developed by Anderson and Borg for highly recyclable materials and products [4]. The concern is how to calculate "how many times material is used in a society from cradle (resource extraction) to grave (landfill)". Nyland et al. [5] described a mathematical geometric progression approach that can be used to expand the system boundaries and allows for recycling a given number of times. They calculated the 'total quantity material function, t (kg)' depending on the recycling rate and the number of times the material is recycled. It was suggested that, if a mass fraction ϕ of a given material is recycled a number of times, n, the total quantity material function, t, can be calculated with **Eq. 1**.

$$t = m + m\,\phi_1 + m\,\phi_1\,\phi_2 + m\,\phi_1\,\phi_2\,\phi_3 + \ldots + m\,\phi_1\,\phi_2\,\phi_3 \ldots \phi_n = m\,(1 + \phi_1 + \phi_1\,\phi_2 + \phi_1\,\phi_2\,\phi_3 + \ldots + \phi_1\,\phi_2\,\phi_3 \ldots \phi_n) \quad \text{(Eq. 1)}$$

where m is the input of the material in the primary product. In this method, how many times a material is used from cradle to grave is expressed in the parenthesis of Eq. 1. If n approaches infinity and the mass fractions are the same in each recycling loop, Eq. 1 can be described as:

$$t = m\,(1/(1 - \phi)) \quad \text{(Eq. 2)}$$

In this case, how many times a material is used from cradle to grave is expressed as $1/(1 - \phi)$.

This approach is useful for simple recycling models, such as shown in **Fig. 1**, but not applicable to complicated material flows. In most advanced countries, where recycling of materials from post-consumer products are enhanced, the material flows are usually complicated with many loops. So, this approach seems not applicable.

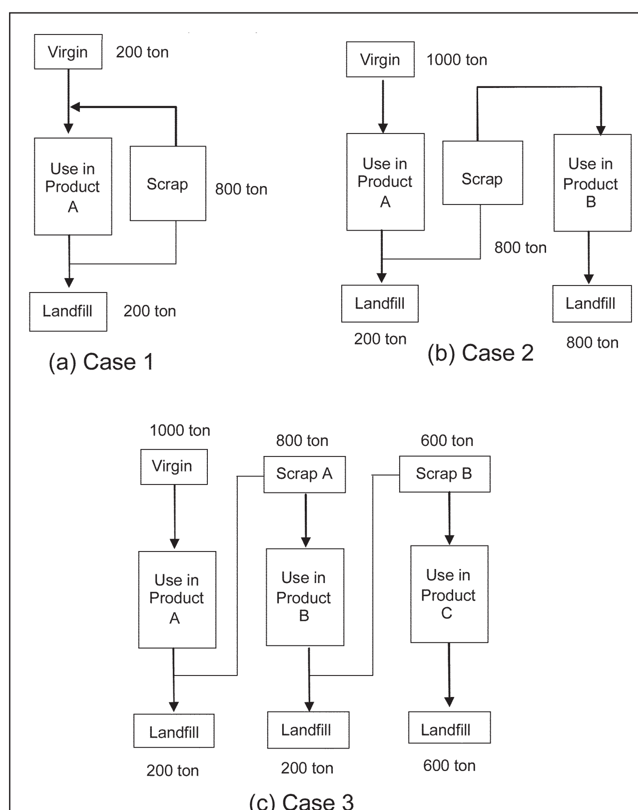

Fig. 1 (a)–(c): Examples of material flow with recycling and states of the element of a material in society

Ekvall suggested that how many times a material is used from cradle to grave could be calculated with **Eq. 3** [6].

$$n = 1/(1-r) \qquad \text{(Eq. 3)}$$

Here r is the rate of collection for recycling of the material from post-consumer products defined by **Eq. 4**.

$$r = c/(c + d) \qquad \text{(Eq. 4)}$$

where c is the amount of recycling of the material from post-consumer products, and d is the amount of the discarded materials from post-consumer products. This method works even if the flow of material is complicated, such as shown in **Fig. 2**. This method is effective when r is constant over a long time and there is a consistency between inflow and outflow of the material to and from the society.

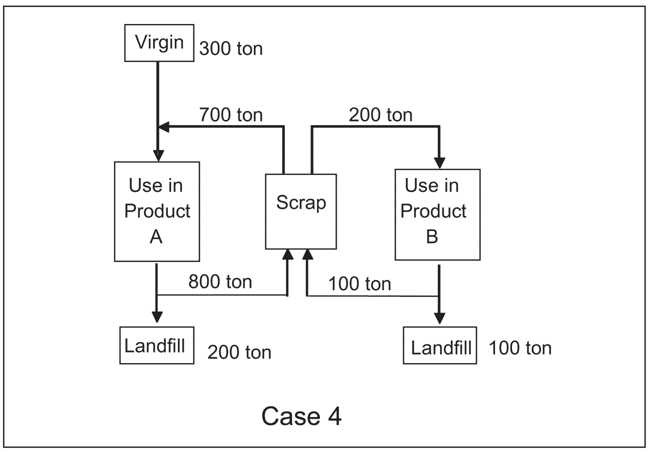

Fig. 2: Examples of material flow and states of the element of a material in society. In a case where closed and open cycle recycling loops exist

Because there is fluctuation in production, discard and recycling of materials in each year in a society, the flow of a material usually has a dynamic aspect. In addition, many products have long lifetime, so, some materials stay in the products for many years in a society. There is always accumulation (or release) of the material to and from the society, as it is shown in **Fig. 3**. Thus, the authors have suggested that a probabilistic method should be applied to estimate how the material is recycled or discarded from post-consumer products.

The purpose of this paper is to develop a methodology to calculate the average number of times a material is used in a society from cradle to grave. Our methodology is based on Markov chain model [7], a probabilistic method using matrix-based numerical analysis, which is effective when state-transitions of material are constant over a long time. Our method is an alternative method to derive the average number of times material is used; this value could be used for allocation of environmental burdens of virgin material as well as an indicator for assessing the state of material use in a certain year, based on the flow of material in that year.

This paper focuses on recycling of metals, especially, steel and aluminum, whose recycling processes are mainly based on re-melting rather than electro-refining. Recycling of other metals, such as copper, is not the focus of this paper, because these metals are recovered by electro-refining, which can be treated as closed-loop recycling.

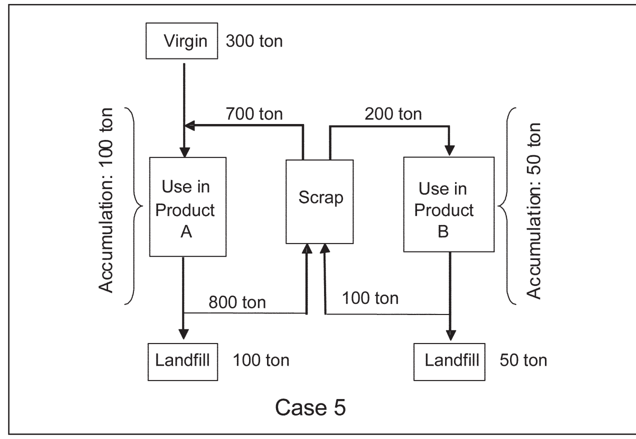

Fig. 3: Typical examples of material flow and states of the element of a material in society. In a case where there is the accumulation of materials in a society

1 Methodology to Analyze the Average Number of Times a Material is Used During its Life Cycle

1.1 Overview of methodology

A material takes various forms in a society, and the probability that the element of material in one state will become in another state depends on the processes involved between the states. The value will be constant as long as there are no changes in processes. A structure in which the probability of becoming the next state is uniquely determined by the current form is called Markovian, and such phenomena can be analyzed by the Markov chain model [7]. The methodology developed in this paper employs the Markov chain model and assumes that when the element of material makes the transition from one state to the next, the transition is uniquely determined on the basis of a probability.

The methodology consists of the following procedure:

1) Modeling the status of residence of a material and its flow in a society
2) Preparation of a state-transition table
3) Preparation of a transition probability matrix
4) Calculating the average number of times that a material is used in products in a society

1.2 Modeling the status of residence of a material and its flow in a society

First, the status of residence of a material is modeled based on the material flow in a society. In this paper, typical examples of material flow of the status of residence of a material and its flow in a society investigated. These were shown in Figs 1–3, Case 1, Case 2, Case 3, Case 4 and Case 5.

All these are hypothetical examples, just to explain how the methodology works. Case 1 shows a case of closed loop recycling. 1,000 ton of a material is used in a product 'A'.

800 ton of the material is recycled at the end of life of the product and used as a resource for the same product. So, actually, 200 ton of a virgin material is produced and 200 ton of the material goes into landfill. Case 2 shows a case of open loop recycling. 1,000 ton of a material is produced and used in a product 'A'. 800 ton of the material is recycled at the end of life of the product and used as a resource for another product 'B'. At the end of life of the product 'B', the material is not recycled and goes into landfill. Case 3 shows a case of cascade open loop recycling. The flow of the material is same with Case 2, but 600 ton of the material is further recycled at the end of life of the product 'B' and used as a resource for another product 'C'.

Case 4 shows a case where both closed loop and open loop recycling exist in a material flow. 1,000 ton of a material is used in a product 'A'. 800 ton of the material is recycled at the end of life of the product, and the other 200 ton is landfilled. On the other hand, 200 ton of the material is used in another product 'B'. At the end of life of the product 'B', 100 ton of the material is recycled and the other 100 ton goes into landfill. In total, 900 ton of scrap is generated, in which 700 ton is used as a resource for another product 'A' and the other 200 ton is used as a resource for another product 'B'. So, actually, 300 ton of a virgin material is produced and 200 ton of the material goes into landfill at the end of life of product 'A', and the rest 100 ton of material goes into landfill at the end of life of product 'B'. It can be confirmed that there is a consistency between the amount of virgin material production and that goes into landfill.

Case 5 shows a case where accumulation of a material exists in a material flow. The flow of the material is almost same with Case 4, but there is accumulation of the material, 100 ton in product 'A' and 50 ton in product 'B'. So, actually, 300 ton of a virgin material is produced, 150 ton is accumulated in the state of products in the society, and the rest 150 ton goes into landfill at the end of life of products.

1.3 Preparation of a state-transition table

Second, a state-transition table is to be created based on the material flow. **Figs. 4 (a)–(e)** are the state-transition tables that have been created for the material flows, Cases 1–5 shown in Figs. 1–3, respectively.

The state-transition table shows how much of the element of material goes from the states shown in rows to those shown in columns. Let us take Fig. 4 (e) (Case 5) as an example. Line 1 shows that 300 ton of virgin material goes to Product A. Line 2 shows the amounts of the material in Product A that goes to other states, which 800 ton goes into scrap and 100 ton goes into landfill. Line 3 shows the amounts of scraps that are recycled as a resource for Product A (700 ton) and Product B (200 ton). Line 4 shows 100 ton of material that is used in Product B goes into scrap and the other 50 ton goes into landfill. Lines 5 gives the amounts of materials in landfill that change to other states, but all elements are zero because landfilled materials remain in the states.

(a) Case 1	Virgin	Product A	Scrap	Landfill	Total
Virgin	0	200	0	0	**200**
Product A	0	0	800	200	**1000**
Scrap	0	800	0	0	**800**
Landfill	0	0	0	0	**0**

(b) Case 2	Virgin	Product A	Scrap	Product B	Landfill	Total
Virgin	0	1000	0	0	0	**1000**
Product A	0	0	800	0	200	**1000**
Scrap	0	0	0	800	0	**800**
Product B	0	0	0	0	800	**800**
Landfill	0	0	0	0	0	**0**

(c) Case 3	Virgin	Product A	Scrap A	Product B	Scrap B	Product C	Landfill	Total
Virgin	0	1000	0	0	0	0	0	**1000**
Product A	0	0	800	0	0	0	200	**1000**
Scrap A	0	0	0	800	0	0	0	**800**
Product B	0	0	0	0	600	0	200	**800**
Scrap B	0	0	0	0		600	0	**600**
Product C	0	0	0	0	0	0	600	**600**
Landfill	0	0	0	0	0	0	0	**0**

(d) Case 4	Virgin	Product A	Scrap	Product B	Landfill	Total
Virgin	0	300	0	0	0	**300**
Product A	0	0	800	0	200	**1000**
Scrap	0	700	0	200	0	**900**
Product B	0	0	100	0	100	**200**
Landfill	0	0	0	0	0	**0**

(e) Case 5	Virgin	Product A	Scrap	Product B	Landfill	Total
Virgin	0	300	0	0	0	**300**
Product A	0	0	800	0	100	**900**
Scrap	0	700	0	200	0	**900**
Product B	0	0	100	0	50	**150**
Landfill	0	0	0	0	0	**0**

Fig. 4 (a)–(e): State-transition tables for the material flows, Case 1–5

1.4 Preparation of a transition probability matrix

Third, a transition probability matrix, **A**, is to be prepared, which has its components calculated with **Eq. 5**.

$$a_{ij} = x_{ij} / X_i, \ (X_i \neq 0) \\ a_{ij} = 0, \qquad (X_i = 0) \tag{5}$$

Here, x_{ij} corresponds the each element of the state-transition table explained in previous chapter and $\mathbf{X}_i$, is the sum of each row in the state-transition table. Elements a_{ij} in matrix **A** represents the probabilities of the material's transition from state i to state j. Thus, matrix **A** is a transition probability matrix showing the probabilities that state i to which element of material belongs will become state j by means of one transition. **Fig. 5** (a)–(e) are the transition probability matrices that have been created for the material flows, Cases 1–5 shown in Figs. 1–3, respectively.

1.5 Calculating the average number of times a material is used in products in a society

A virgin material produced and used in products later becomes scrap after the product lifetime expires; it is consumed as re-input into society. In actuality, the production/consumption structure of the material is different when the material is input and re-input, and the state-transition probabilities are likewise different. Nevertheless, this methodology assumes that the state-transition probability is constant and does not vary with time; in other words, no matter how many times the material is reused, it is always input into society with the same consumption structure, and the transitions of its states happen according to the same probabilities.

As it has been already explained, the probability of a material in initial state i becoming in state j by means of one transition is expressed as a_{ij}, the (i,j)-element of matrix **A**. Let $\mathbf{A}^k$ denote the matrix that is defined by multiplying **A** by itself k times, and $[\mathbf{A}^k]_{ij}$ the (i,j)-element of $\mathbf{A}^k$. If the state-transition probability is invariable, the probability, $P_{ij}(k)$, of a material in initial state i becoming in state j through k transitions is given by **Eq. 6**:

$$P_{ij}(k) = \left[A^k\right]_{ij}, \qquad (j \notin W) \tag{6}$$

where W is the set of states of the discarded, i.e. landfill. Eq. 6 is known as the Chapman-Kolmogorov equation in the literature of stochastic processes; see, e.g., reference [8] for details.

The average number of transitions in state u, the state of residence in a product, can be expressed as N_{su} where N_{su} is determined by **Eq. 7**.

$$\begin{aligned} N_{su} &= \lim_{n\to\infty} \sum_{k=0}^{n} P_{su}(k) \\ &= \left[\lim_{n\to\infty} \sum_{k=0}^{n} A^k\right]_{su} \\ &= \left[(I-A)^{-1}\right]_{su}, \qquad (u \notin W) \end{aligned} \tag{7}$$

Here, s means the initial state and I means the identity matrix. The third equality holds because the transition probability matrix satisfies the condition of Solow, corollary of theorem 1 [9]. This equation transformation is frequently used in the input-output economics.

(a) Case 1	Virgin	Product A	Scrap	Landfill
Virgin	0.00	1.00	0.00	0.00
Product A	0.00	0.00	0.80	0.20
Scrap	0.00	1.00	0.00	0.00
Landfill	0.00	0.00	0.00	0.00

(b) Case 2	Virgin	Product A	Scrap	Product B	Landfill
Virgin	0.00	1.00	0.00	0.00	0.00
Product A	0.00	0.00	0.80	0.00	0.20
Scrap	0.00	0.00	0.00	1.00	0.00
Product B	0.00	0.00	0.00	0.00	1.00
Landfill	0.00	0.00	0.00	0.00	0.00

(c) Case 3	Virgin	Product A	Scrap	Product B	Scrap B	Product C	Landfill
Virgin	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Product A	0.00	0.00	0.80	0.00	0.00	0.00	0.20
Scrap	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Product B	0.00	0.00	0.00	0.00	0.75	0.00	0.25
Scrap B	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Product C	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Landfill	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(d) Case 4	Virgin	Product A	Scrap	Product B	Landfill
Virgin	0.00	1.00	0.00	0.00	0.00
Product A	0.00	0.00	0.80	0.00	0.20
Scrap	0.00	0.78	0.00	0.22	0.00
Product B	0.00	0.00	0.50	0.00	0.50
Landfill	0.00	0.00	0.00	0.00	0.00

(e) Case 5	Virgin	Product A	Scrap	Product B	Landfill
Virgin	0.00	1.00	0.00	0.00	0.00
Product A	0.00	0.00	0.89	0.00	0.11
Scrap	0.00	0.78	0.00	0.22	0.00
Product B	0.00	0.00	0.67	0.00	0.33
Landfill	0.00	0.00	0.00	0.00	0.00

Fig. 5 (a)–(e): Transition probability matrices for the material flows, Case 1–5

In Eq. 7, the initial state (state in virgin material) is included in the number of transitions, but if the initial state is to be excluded, calculation is possible by starting the range of k at 1.

Thus, the total average number of times a material used in products from the initial state s until ultimately being landfilled through an unlimited number of transitions can be derived by **Eq. 8**.

$$t_s = \sum_{u \in U} N_{su} \tag{8}$$

where U means product categories.

2 Results and Discussions

Fig. 6 (a)–(e) and **Fig. 7 (a)–(e)** show the matrices (**I–A**) and $(\mathbf{I}-\mathbf{A})^{-1}$ obtained from the state-transition tables shown in Figs. 4 (a)–(e), respectively.

The matrix $(\mathbf{I}-\mathbf{A})^{-1}$ shows how many times a virgin material is used in products until all of the element is ultimately landfilled. See the values with asterisk in Figs. 7 (a)–(e). **Table 1** is the summary of the number of times a virgin material is used in products for each case.

Table 1: The summary of the number of times a virgin material is used in products for each case

Case 1	5.0
Case 2	1.8
Case 3	2.4
Case 4	4.0
Case 5	6.54

In Case 1, the elements of virgin material are used 5.0 times in average before they are landfilled. It can be confirmed that only 200 tons of virgin material are produced for Product 1 although Product 1 consists of 1000 tons of material. So, the total quantity material function of the virgin material should be 5. In Case 2, the elements of virgin material are used 1.0 times in Product A and 0.8 times in Product B. So, they are used 1.8 times in average before they are landfilled. It is also apparent because all the elements of the virgin material are used once in Product A and 80% of them go into scrap and are used in Product B. After they are used in Product B, all of the elements go into landfill. In Case 3, the elements of virgin material are used 2.4 times in average before they are landfilled. This result can be confirmed by Eq. 1. In Case 4 and 5, the material flow is more complicated. So, one can not know at sight how many times the elements of virgin material are used in products before they are ultimately landfilled. With our methodology, it was calculated that, in Case 4, the elements of virgin material are used 3.33 times in Product A and 0.67 times in Product B, resulting in 4.0 times in total. On the other hand, it was calculated that, in Case 5, the elements of virgin material are used 5.31 times in Product A and 1.23 times in Product B, resulting in 6.54 times in total.

These results were compared with those calculated by other methodology. **Table 2** is the summary of the number of times a virgin material is used in products for each case, calculated by Eq. 3 and 4.

As it was shown in Table 2, the results for Cases 1–4 are the same with Table 1. There is a difference in the result for Case 5.

(a) Case 1	Virgin	Product A	Scrap	Landfill
Virgin	1.00	–1.00	0.00	0.00
Product A	0.00	1.00	–0.80	–0.20
Scrap	0.00	–1.00	1.00	0.00
Landfill	0.00	0.00	0.00	1.00

(b) Case 2	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	–1.00	0.00	0.00	0.00
Product A	0.00	1.00	–0.80	0.00	–0.20
Scrap	0.00	0.00	1.00	–1.00	0.00
Product B	0.00	0.00	0.00	1.00	–1.00
Landfill	0.00	0.00	0.00	0.00	1.00

(c) Case 3	Virgin	Product A	Scrap	Product B	Scrap B	Product C	Landfill
Virgin	1.00	–1.00	0.00	0.00	0.00	0.00	0.00
Product A	0.00	1.00	–0.80	0.00	0.00	0.00	–0.20
Scrap	0.00	0.00	1.00	–1.00	0.00	0.00	0.00
Product B	0.00	0.00	0.00	1.00	–0.75	0.00	–0.25
Scrap B	0.00	0.00	0.00	0.00	1.00	–1.00	0.00
Product C	0.00	0.00	0.00	0.00	0.00	1.00	–1.00
Landfill	0.00	0.00	0.00	0.00	0.00	0.00	1.00

(d) Case 4	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	–1.00	0.00	0.00	0.00
Product A	0.00	1.00	–0.80	0.00	–0.20
Scrap	0.00	–0.78	1.00	–0.22	0.00
Product B	0.00	0.00	–0.50	1.00	–0.50
Landfill	0.00	0.00	0.00	0.00	1.00

(e) Case 5	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	–1.00	0.00	0.00	0.00
Product A	0.00	1.00	–0.89	0.00	–0.11
Scrap	0.00	–0.78	1.00	–0.22	0.00
Product B	0.00	0.00	–0.67	1.00	–0.33
Landfill	0.00	0.00	0.00	0.00	1.00

Fig. 6 (a)–(e): Matrices (**I–A**) for Case 1–5

(a) Case 1	Virgin	Product A	Scrap	Landfill
Virgin	1.00	5.00 *	4.00	1.00
Product A	0.00	5.00	4.00	1.00
Scrap	0.00	5.00	5.00	1.00
Landfill	0.00	0.00	0.00	1.00

(b) Case 2	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	1.00 *	0.80	0.80 *	1.00
Product A	0.00	1.00	0.80	0.80	1.00
Scrap	0.00	0.00	1.00	1.00	1.00
Product B	0.00	0.00	0.00	1.00	1.00
Landfill	0.00	0.00	0.00	0.00	1.00

(c) Case 3	Virgin	Product A	Scrap	Product B	Scrap B	Product C	Landfill
Virgin	1	1 *	0.8	0.8 *	0.6	0.6 *	1
Product A	0	1	0.8	0.8	0.6	0.6	1
Scrap	0	0	1	1	0.75	0.75	1
Product B	0	0	0	1	0.75	0.75	1
Scrap B	0	0	0	0	1	1	1
Product C	0	0	0	0	0	1	1
Landfill	0	0	0	0	0	0	1

(d) Case 4	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	3.33 *	3.00	0.67 *	1.00
Product A	0.00	3.33	3.00	0.67	1.00
Scrap	0.00	2.92	3.75	0.83	1.00
Product B	0.00	1.46	1.88	1.42	1.00
Landfill	0.00	0.00	0.00	0.00	1.00

(e) Case 5	Virgin	Product A	Scrap	Product B	Landfill
Virgin	1.00	5.31 *	5.54	1.23 *	1.00
Product A	0.00	5.31	5.54	1.23	1.00
Scrap	0.00	4.85	6.23	1.38	1.00
Product B	0.00	3.23	4.15	1.92	1.00
Landfill	0.00	0.00	0.00	0.00	1.00

Fig. 7 (a)-(e): Matrices $(\mathbf{I}-\mathbf{A})^{-1}$ obtained for Case 1–5

Table 2: The summary of the number of times a virgin material is used in products for each case, calculated by Eq. 3 and 4

	c	*d*	*r*	Result
Case 1	800	200	0.800	5.0
Case 2	800	1000	0.444	1.8
Case 3	1400	1000	0.583	2.4
Case 4	900	300	0.750	4.0
Case 5	900	150	0.857	7.0

As it was mentioned before, the material flow in Case 5 is not static, and there is accumulation of the material. In this case, the accuracy of the results obtained by these methods depends on the situation. Our methodology adopts a probabilistic method to estimate how the material of a state goes into the next state. So, it could be mentioned that the results obtained by our methodology are valid if state-transitions of material are constant over a long time. On the other hand, the results obtained by other method (Eq. 3 and 4) are valid if recycling rate is constant over a long time.

3 Conclusion

In this paper, a method was developed to calculate the average number of times a material is used in a society from cradle to grave. Our methodology is based on Markov chain model, a probabilistic method using matrix-based numerical analysis. A major feature of this method is that it creates transition probability matrices for a material from the way in which the material is produced, consumed, and recycled, making it possible to simply elicit indicators that assess the status of material use in products in society.

This methodology is effective when state-transitions of material are constant over a long time. Our method is an alternative method to derive the average number of times material is used.

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